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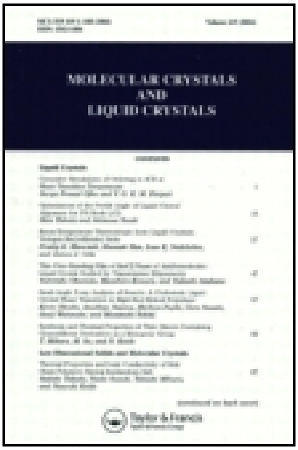

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Soliton Solutions for Orientation Waves in Nematics

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Soliton Solutions for Orientation Waves in Nematics†

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The propagation of orientation waves in nematics placed in a constant magnetic field and excited by pulsed magnetic fields of finite sizes is studied. It is shown that in the geometries of the experiment and allowing for the emergence of back-flow, formation of soliton solutions is simplified in comparison with the case when back-flow is absent.

The non-linear dynamics of liquid crystals have recently been extensively investigated. The interest in these problem is accounted for in part by the satisfactory state of development of linearised treatments of liquid crystal dynamics. Further motivation has been provided by experimental results which have recently been obtained in a number of laboratories and which cannot be explained in the framework of these linear theories.

At present a great number of liquid crystals, with widely differing physical properties have been synthesized. This wide range of variations of their characteristic parameters provides a unique opportunity for studying non-linear effects.

The dynamics of the director in a nematic liquid crystal in a magnetic field in the linear approximation have been dealt with by a number of authors and the results are well known (see refs. 1, 2 and references therein).

In ref. 3, the high-frequency dynamics of the nematic director have been computed in a magnetic field. However, in this work the authors have confined themselves to the case of homogeneous motion, i.e.,

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they have not touched upon spatial effects. It is nevertheless of interest to study the dynamics of the director with spatial dependences taken into account, in particular since the experimental data point to their importance. In ref. 4, for example, the authors have experimentally investigated propagation of the director waves in a nematic under uniform shear. It has been found that in the sample in the direction of the flow there occur alternating domains with the orthogonal orientation of the director. A qualitative theory constructed for the description of the results of this experiment[5] accounts for the observed picture by emergence of soliton solutions in the director wave propagation. This explanation seems fairly reasonable.

Possibilities of soliton solutions for orientation waves in membranes have been qualitatively discussed.[6] The author has solved[7] the problem of propagation of orientation twist waves in a nematic placed in a constant uniform magnetic field H and excited by a pulsed magnetic field h in the geometry shown in Figure 1a. It has been shown that for certain values of the parameters of the system soliton solutions may occur as kinks, antikinks, or breathers depending on the initial conditions of the problem. For this problem to be solved it is important that in the above geometry there is no bulk motion of the nematic, i.e., there is no back-flow affecting the motion of the director.

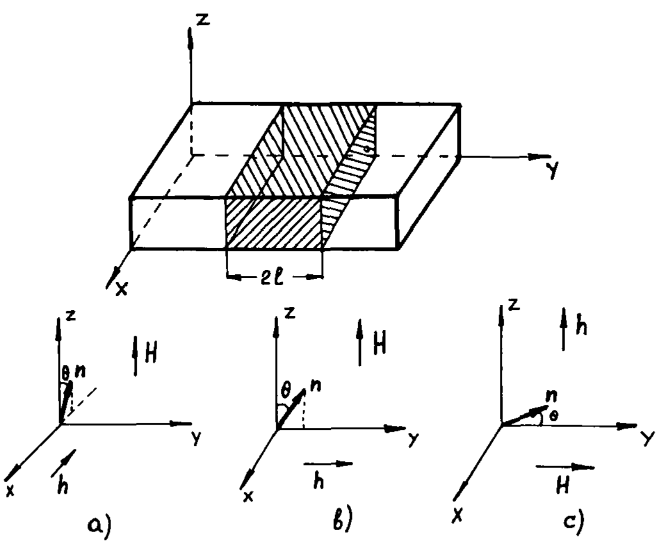

FIGURE Possible geometries of the experiment and the form of the sample.

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Another important feature of this study is the assumption that the parameter $\gamma_1/H(\chi_a J)^{1/2}$, where γ_1 is the viscosity coefficient, χ_a is the anisotropic part of the magnetic susceptibility, J is the inertia moment of the director, is small. This assumption imposes certain constraints upon the quantities H and J at which a soliton solution is existent.

This paper aims at studying propagation of orientation waves in geometries shown in Figures 1b, 1c, i.e., in the cases when back flows must necessarily be taken into account. A similar linear problem has been solved,[8,9] but the non-linear case investigated here also differs from that[8,9] in the boundary conditions considered.

We assume that the coupling of the nematic to the boundary surfaces of the crystal is small and that the size of the sample is sufficiently large, so that the effects caused by these surfaces influence the orientation of the director and the motion of molecules only in a narrow region near the boundaries. These assumptions render the problem uniform with respect to the axes Y and X.

Equations for the dynamics of a nematic, derived by Ericksen and Leslie[10] are well known. For the director **n** the equation of motion is written as

$$J\frac{\mathrm{d}}{\mathrm{dt}}\left[\mathbf{n}\frac{\mathrm{d}\mathbf{n}}{\mathrm{dt}}\right] = [\mathbf{nf}] - [\mathbf{nR}] \tag{1}$$

where **f** is a volume force bringing **n** to the equilibrium value; **R** is a dissipative force.

In the derivation of this equation it has been borne in mind that $\mathbf{n}^2 = 1$ and that the director changes only its orientation.

The forces **f** and **R** are determined by the expressions [1]

$$\mathbf{f} = \mathrm{K}_1\nabla(\mathrm{div}\mathbf{n}) - \mathrm{K}_2[(\mathbf{n}\ \mathrm{rot}\mathbf{n})\mathrm{rot}\mathbf{n} + \mathrm{rot}(\mathbf{n}(\mathbf{n}\ \mathrm{rot}\mathbf{n}))] +$$

$$+ \mathrm{K}_3\{\mathrm{rot}[\mathbf{n}[\mathbf{n}\ \mathrm{rot}\mathbf{n}]] - [\mathrm{rot}\mathbf{n}[\mathbf{n}\ \mathrm{rot}\mathbf{n}]]\} + \chi_a(\mathbf{Hn})\mathbf{H}, \tag{2}$$

$$R_i = \gamma_1 N_i + \gamma_2 n_j A_{ji},$$

where

$$A_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right),\ \mathbf{N} = \frac{\mathrm{d}\mathbf{n}}{\mathrm{dt}} - \frac{1}{2}[\mathrm{rot}\mathbf{v}\ \mathbf{n}],$$

v is the velocity of the substance, γ_1 and γ_2 are viscosity coefficients, K_i are Frank constants.

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By virtue of the uniformity of the problem and the incompressibility conditions div $\mathbf{v} = 0$, the only non-zero components of the tensor A_{ij} are $A_{yz} = A_{zy} = 1/2(\partial v_z/\partial y)$.

Writing down the director $\mathbf{n}$ in the form $n = (0, \cos\theta, \sin\theta)$ in the case of Figure 1b and $n = (0, \sin\theta, \cos\theta)$ in the case of Figure 1c, where θ is the angle of the deviation of the director from its equilibrium, and confining ourselves to one-constant approximation, we find that

$$J\ddot{\varphi} - K\varphi'' - \chi_a[(h^2 - H^2)\sin\varphi + 2Hh\cos\varphi] + \gamma_1\dot{\varphi} + \frac{\partial v_z}{\partial y}(\gamma_1 + \gamma_2\cos\varphi) = 0 \quad (3')$$

$$J\ddot{\varphi} - K\varphi'' - \chi_a[(h^2 - H^2)\sin\varphi + 2Hh\cos\varphi] + \gamma_1\dot{\varphi} - \frac{\partial v_z}{\partial y}(\gamma_1 - \gamma_2\cos\varphi) = 0 \quad (3'')$$

respectively, where $\varphi = 2\theta$.

A qualitative consideration of these equations at small angles φ reveals that the last terms of these equations affect their solutions differently. In the first case $(\partial v_z/\partial y)$ is multiplied by the small coefficient $\gamma_1 + \gamma_2 \ll \gamma_1$, in the second case – by $\gamma_1 - \gamma_2 \approx 2\gamma_1$.

For a more accurate solution let us write out the Navier-Stokes equations for the both cases

$$\rho\frac{\partial v_z}{\partial t} = \frac{1}{4}\frac{\partial}{\partial y} \cdot \left[\frac{\partial\varphi}{\partial t}(\gamma_1 + \gamma_2\cos\varphi) + \frac{\partial v_z}{\partial y}(\gamma_1 + 2\alpha_4 + \alpha_5 + \alpha_6 + 2\gamma_2\cos\varphi)\right], \quad (4')$$

$$\rho\frac{\partial v_y}{\partial t} = \frac{1}{4}\frac{\partial}{\partial y}\left[\sin\varphi\left[\gamma_2\frac{\partial\varphi}{\partial t} + \frac{\partial v_z}{\partial y}(\gamma_2 + \alpha_5 + \alpha_6)\right]\right]$$

$$\rho\frac{\partial v_z}{\partial t} = \frac{1}{4}\frac{\partial}{\partial y} \cdot \left\{(2\alpha_4 + \alpha_5 + \alpha_6 - 2\gamma_2\cos\varphi + \gamma_1)\frac{\partial v_z}{\partial y} - \frac{\partial\varphi}{\partial t}(\gamma_1 - \gamma_2\cos\varphi)\right\}, \quad (4'')$$

$$\rho\frac{\partial v_y}{\partial t} = \frac{1}{4}\frac{\partial}{\partial y}\left\{\sin\varphi\left[(\alpha_5 + \alpha_6 + \gamma_2)\frac{\partial v_z}{\partial y} - \gamma_2\frac{\partial\varphi}{\partial t}\right]\right\}$$

respectively. Here α_i are Leslie coefficients. It is clear that the solution of the system of equations (3) and (4) is a complicated task and can hardly be found analytically. Therefore it is interesting to study at least qualitatively the influence of the last term of Eqs. (3) upon soliton solutions. Below we shall assume that soliton solutions of (3) exist and shall make an attempt at finding the appropriate solutions of Eqs. (4). The substitution of these solutions into (3) will give us an idea of tendencies of the soliton solution behaviour with back-flow taken into account.

The solution[7] of Eq. (1) has been considered for the case of absence of back-flow and in the assumption that the parameter $\gamma_1/H(\chi_a J)^{1/2} \ll 1$.

Eq. (1) is then written as

$$\frac{\partial^2\varphi}{\partial\tau^2} - \frac{\partial^2\varphi}{\partial\xi^2} + \eta\frac{\partial\varphi}{\partial\tau} + \frac{H^2-h^2}{H^2}\sin\varphi - \frac{2h}{H}\cos\varphi = 0 \qquad (5)$$

where $\eta = \gamma_1/H(\chi_a J)^{1/2}$ is the effective damping, $\tau = tH(\chi_a/J)^{1/2}$, $\xi = yH(\chi_a/K)^{1/2}$.

Generally speaking, for nematics the value of η in the region of mean magnetic fields $\sim 10^2-10^3$ Oe and the molecular value of J are not small. However, for fields $\sim 10^4-10^5$Oe, $\gamma_1 \sim 10^{-2}P$, $\chi_a \sim 10^{-6}$ CGSE units, the condition $\eta < 1$ can be fulfilled for $J \sim 10^{-8}$g/cm. The quantity η can become small in the region of the nematic-isotropic liquid transition where the coefficient γ_1 tends to zero.

Conventionally Eq. (5) is linearized and the inertial term $J(\partial^2\varphi/\partial t^2)$ is omitted, since it is asserted that J is determined by molecular characteristics and is small ($J \sim 10^{-14}$g/cm). So far the value of J has not been obtained experimentally.

It is possible to assume that new kinds of nematics with more inertial molecules will be synthesized. In particular, inertial effects may prove important for the study of lyotropic liquid crystals where nematic order is determined by large clusters of molecules.

Initially the velocity of the deviation of the director from its equilibrium is taken to be uniform (Figure 1a) in a limited region of the XZ plane which must have sufficiently sharp boundary in the Y direction. Subsequently, this perturbation will start propagating in the Y direction and the problem will still be effectively uniform if the influence of the boundaries of the sample is ignored.

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If we completely neglect dissipation effects, Eq. (5) reduces to the Sine-Gordon equation with the initial conditions:

$$\varphi(\xi,0) = 0, \frac{\partial\varphi}{\partial\xi}(\xi,0) = 0, \frac{\partial\varphi}{\partial\tau}(\xi,0) = 2F_0(\xi).$$

Then by the inverse scattering method[11,12] depending on the initial conditions and the parameters of the system, solutions of Eq. (5) can be found. In particular, it has been shown that at

$$2hl(h^2 - H^2)^{1/2}\,\mathrm{sh}\Omega t_p > \pi,$$

where $\Omega = [(h^2 - H^2)\chi_a/J]^{1/2}$ and t_p is the duration of the pulse of the exciting field h; in the system there emerge solitons of the types of breathers and kink-antikink pairs. A number of emerging solitons and their characteristics (propagation velocity, characteristic size) have also been calculated as functions of the parameters of the system.

In particular, in the case when kink-antikink pairs are created, the solution is:

$$\varphi_n = 4\ \mathrm{arc\ tg}\ \exp\left[-\epsilon\frac{\xi - \xi_{on} - V_n\tau}{(1 - V_n^2)^{1/2}}\right], \tag{6}$$

where ϵ sets the form of the soliton (kink at $\epsilon = -1$, antikink at $\epsilon = 1$), n is the number of a soliton, $V_n = t_n^{-1}(t_n^2 - 1)^{1/2}$ is the velocity of the n-th soliton, ξ_{on} is the initial coordinate of a soliton and t_n is the appropriate solution of the equation:

$$\mathrm{tg}\ l(F_0^2 - t^2)^{1/2} = -(F_0^2 - t^2)^{1/2}/t.$$

The influence of the dissipative term $\eta(\partial\varphi/\partial\tau)$ can then be taken into account according to the perturbation theory.[13] The results of the work[7] have been briefly covered here, since, as will be shown below, in the presence of back-flow the problem is effectively reduced to the solution of Eq. (5).

We shall assume, as before,[7] that $\eta < 1$. Apparently, if the derivative $(\partial v_z/\partial\xi)$ has no singularities and by the order of magnitude coincides with $(\partial\varphi/\partial\tau)$, the solution for φ will coincide with the solution obtained[7] with the accuracy up to corrections determined by means of the perturbation theory.

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The scheme of further speculations is as follows: assuming that the dissipative term in the first equation of the system (3′–3″) is small in comparison with the remaining terms, we shall obtain the expression for $V_z(y,\tau)$ and by inserting it into the equation for φ, we shall make sure that our assumption is justified.

Consider the system of equations (3′–3″) at time t, close to zero, when $\varphi \ll 1$. In terms of dimensionless variables ξ, τ, it is

$$\frac{\partial^2\varphi}{\partial\tau^2} - \frac{\partial^2\varphi}{\partial\xi^2} + \varphi + \eta\left(\frac{\partial\varphi}{\partial\tau} + \beta_i\frac{\partial u_z}{\partial\xi}\right) = 0,$$

$$\delta\frac{\partial u_z}{\partial\tau} = \frac{\partial}{\partial\xi}\left(\mu_i\frac{\partial u_z}{\partial\xi} + \beta_i\frac{\partial\varphi}{\partial\tau}\right), \tag{7}$$

$$\delta\frac{\partial u_y}{\partial\xi} = \frac{\partial}{\partial\xi}\left[\varphi\left(\frac{2\alpha_6}{\gamma_1}\frac{\partial u_z}{\partial\xi} + \upsilon_i\frac{\partial\varphi}{\partial\tau}\right)\right].$$

Here $u = v(J/K)^{1/2}$, $\beta_{1,2} = (\gamma_2 \pm \gamma_1/\gamma_1)$, $\mu_{1,2} = 2(\alpha_4 + \alpha_5 + \alpha_3 \pm \gamma_2)/\gamma_1$, $\upsilon_{1,2} = \pm\gamma_2/\gamma_1$, $\delta = 4\,\rho K/\gamma_1 H\sqrt{\chi_a J}$.

The second equation of the system can be written as an inhomogeneous diffusion equation

$$\frac{\partial u_z}{\partial\tau} = a^2\frac{\partial^2 u_z}{\partial\xi^2} + f(\xi, \tau) \tag{8}$$

where $a^2 = (\mu_i/\delta)$ and the source term $f(\xi, \tau) = (\beta_i/\delta)(\partial^2\varphi/\partial\xi\partial\tau)$. The solution of Eq. (8) with the appropriate initial condition $u_z(\tau = 0) = 0$ and boundary conditions $u_z(\pm\infty, \tau) = 0$ is:

$$u_z(\xi,\tau) = \int_0^\tau\int_{-\infty}^{\infty} f(\xi',\tau')\exp\left[-\frac{(\xi' - \xi)^2}{4a^2(\tau - \tau')}\right]\frac{d\xi' d\tau'}{2a\sqrt{\pi(\tau - \tau')}} \tag{9}$$

For further study it is necessary to find the dependence $(\partial^2\varphi/\partial\xi\partial\tau)$ on a ξ and τ. In the zero approximation with respect to η (i.e., when the dissipative term is neglected) the first equation of the system is a well-known telegraph equation with the initial conditions:

$$\varphi(\xi, 0) = 0$$

$$\frac{\partial\varphi}{\partial\tau}(\zeta, 0) = \begin{cases} 2F_0 & \text{at } -\ell \leqslant \xi \leqslant \ell \\ 0 & \text{at remaining } \xi \end{cases}$$

The solution of this equation satisfying the indicated initial conditions can be easily found and has the form

$$\varphi = \frac{2F_0}{\pi}\int_{-\infty}^{\infty} \frac{\sin k\ell \sin(\tau\sqrt{k^2+1})}{k\sqrt{k^2+1}} \exp(ik\xi)dk$$

As we are interested in the quantity $(\partial^2\varphi/\partial\tau\partial\xi)$, we first calculate

$$\frac{\partial\varphi}{\partial\xi} = \frac{2_iF_0}{\pi}\int_{-\infty}^{\infty} \frac{\sin k\ell \sin(\tau\sqrt{k^2+1})}{\sqrt{k^2+1}} \exp(ik\xi)dk \qquad (10)$$

Performing the integration in Eq. (10) and differentiating the result over τ, we get for $\tau \ll \ell$

$$\frac{\partial^2\varphi}{\partial\tau\partial\xi} = \begin{cases} \dfrac{-F_0\tau J_1(\sqrt{\tau^2-(\ell+\xi)^2})}{\sqrt{\tau^2-(\ell+\xi)^2}} & \text{at } -\ell-\tau<\xi<-\ell+\tau \\[2ex] \dfrac{F_0\tau J_1(\sqrt{\tau^2-(\ell-\xi)^2})}{\sqrt{\tau^2-(\ell-\xi)^2}} & \text{at } \ell-\tau<\xi<\ell+\tau \\[2ex] 0 & \text{at remaining } \zeta \end{cases} \qquad (11)$$

here J_1 is a Bessel function of the first kind of order 1. This form of $(\partial^2\varphi/\partial\tau\, \partial\xi)$ is a consequence of implying that the initial condition for $(\partial\varphi/\partial\tau)$ at $\tau = 0$ has been set in the form of a step-like function. The "signal" propagates on either side from the edges with a unit velocity and occupies a region of the size 2τ, symmetric with respect to the points $\pm\ell$.

Thus, variation of $\partial\varphi/\partial\tau$ takes place only in the above-mentioned regions. Between them the initial value of $(\partial\varphi/\partial\tau) = 2F_0$ is retained, whereas in the regions outside them $(\partial\varphi/\partial\tau) = 0$.

In principle, the insertion of Eq. (11) into Eq. (9) and subsequent differentiation over ξ, yields the solution of the problem. However, for our purposes qualitative considerations are sufficient. As follows from the form of $(\partial^2\varphi/\partial\tau\partial\xi)$, the characteristic time of variation of the source τ_0 in the Eq. (8) is of the order of ℓ; the characteristic time over which the velocity u_z is established is: $\tau_2 \sim \ell^2/a^2$. Assuming that $\ell \sim 1$, which corresponds to soliton creation, and taking into account that $a \gg 1$ even at $\eta \lesssim 1$, we get $\tau_2 \ll \tau_0$. This means that the velocity u_z immediately adjusts to variations of the source, i.e.,

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inertia is absent. Hence it follows from Eq. (8) that $(\partial u_z/\partial \tau) \ll f(\xi, \tau)$ and, consequently

$$\frac{\partial u_z}{\partial \xi} \approx -\frac{\beta_i}{\mu_i}\frac{\partial \varphi}{\partial \tau} \tag{12}$$

Inserting this expression into Eq. (7) for φ, we find that the dissipative term has the form $\eta_{eff}\,(\partial\varphi/\partial\tau)$
where

$$\eta_{eff} = \left(1 - \frac{\beta_i^2}{\mu_i}\right)\eta.$$

So far we have treated the case $\tau \ll \ell$, which corresponds to the chosen approximation $\varphi \ll 1$. It is possible to obtain expressions, analogous to (11) for $\tau \sim \ell$ also. The fact that in this case Eqs. (3), (3′) will involve sin φ but not φ as in the case of Eq. (7), will only simplify the situation. The fact that smooth functions of φ stand before $(\partial u_z/\partial\xi)$ and $(\partial\varphi/\partial\tau)$ in the Navier-Stokes equations, preserves the validity of the assertion concerning the characteristic times. Therefore as a result, for the effective viscosity coefficient η_{eff} we have

$$\eta_{eff} = \eta\left[1 - \frac{(\gamma_2\cos\varphi \pm \gamma_1)^2}{2(\alpha_4 + \alpha_5 + \alpha_3 \pm \gamma_2\cos\varphi)}\right].$$

Since for all φ $\eta_{eff} \leq \eta$ it is possible to conclude that the presence of back-flow simplifies soliton creation in the system and at $\eta < 1$ the results previously given[7] are applicable to the case under study.

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